of the field: For $K_0 = 0$, Eq. (3.9) splits off from the system (3.2), (3.6)-(3.8); T(x, y, z) and $e_\alpha(x, y, z)$ are determined from the given functions $v_0(z)$ and $b_0(z)$.

The above-proposed scheme is based on rejection of the terms $G_i$ in the system (1.8)-(1.12). The solution can be subsequently refined either by a successive-approximation procedure or by obtaining a Riccati equation for $G_i \neq 0$.

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# ELECTRIC FIELD BUILDUP IN PORE COLLAPSE


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The rapid deformation and fracture of solids gives rise to strong electric fields with a resultant emission of particles, x-rays, and radio-frequency radiation from the fracture surface. The field is generated by the production and separation of point defects and charged dislocations on the shock front [1] and at the tips of growing cracks [2, 3]. In this paper we examine the electric effects arising near cavities and pores which collapse in a shock wave.

Consider a porous dielectric medium. When the material undergoes impact compression, the highest deformation rates occur in the plastic zones localized around cavities and inhomogeneous inclusions [4]. These zones are production sites of point defects and electrically charged dislocations. The defect multiplication rate is proportional to the shear deformation rate dγ/dt. At low concentrations the recombination of point defects can be neglected [1]. In the case of two defect types with opposite charges, the equation of continuity (i = 1, 2) has the form

$$\frac{\partial n_i}{\partial t} + \operatorname{div} \mathbf{j}_i = M\left|\frac{d\gamma}{dt}\right|, \quad \mathbf{j}_i = n_i\mathbf{v} - \delta \operatorname{grad} n_i\nu_i + \frac{\sigma_i}{q_i}\mathbf{E}. \tag{1}$$

Here M is the multiplication constant; $n_i$ and $q_i$ are the point-defect concentration and charge, or the number of dislocations per unit area and the charge per unit dislocation length; the defect current density $\mathbf{j}_i$ has three components, involving the lattice velocity of motion $\mathbf{v}$, the defect displacement relative to the lattice (diffusion), and the drift in the field of strength $\mathbf{E}$; $\nu_i$ is the displacement frequency of a defect over one interatomic distance $\delta$ ($\delta\nu_i$ is the dislocation velocity); $\sigma_i$ is the ionic conductivity.

For rapidly varying loads, $\mathbf{j}_i = n\mathbf{v}_i$ at first approximation. Leaving the defect type unspecified, we substitute this in (1). Assuming the material surrounding the pores is incompressible, i.e., div v = 0, we obtain $n_i \equiv n = n_0 + M\gamma$ in Lagrangian coordinates ($n_0$ is the initial defect concentration).

In the following approximation we seek small corrections $m_i \ll n$. Setting in (1) $n_i = n + m_i$, we get

$$\frac{\partial m_i}{\partial t} + \operatorname{div} m_i\mathbf{v} - \delta^2\Delta n\nu_i + \frac{1}{q_i}\operatorname{div}\sigma_i\mathbf{E} = 0, \tag{2}$$


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where $\sigma_i$ is a function of n. Multiplying (2) by $q_1 = e$ for $i = 1$ and by $q_2 = -e$ for $i = 2$ and summing the results, we obtain the equation of continuity for the current. Adding one of Maxwell's equations, we arrive at the system

$$\partial p/\partial t + \operatorname{div} \mathbf{J} = 0, \ \mathbf{J} = \rho \mathbf{v} - e\delta^2 \operatorname{grad} n (\nu_1 - \nu_2) + (\sigma_1 + \sigma_2)\mathbf{E}, \\ \varepsilon\varepsilon_0 \operatorname{div} \mathbf{E} = \rho, \ \rho = e(m_1 - m_2). \tag{3}$$

Here J is the current density due to both conduction and nonelectric forces, ρ is the electric charge density, and ε is the permittivity of the medium. The expression for J shows that the charge separation is due to the difference in the $\nu_i$ values, which denote the defect "mobility" in the shock wave.

The above approximations can be evaluated by taking into account that in the neighborhood of a pore of radius $a$, $\gamma \sim v/a$ and $E \sim \rho a/(\varepsilon\varepsilon_0)$. The second and third terms in the expression for $\mathbf{j}_i$ are therefore smaller than the first when $\gamma \gg \delta^2\nu/a^2$ and $\gamma \gg m/(n\tau)$ ($\tau \sim \varepsilon\varepsilon_0/\sigma$ is the relaxation time by conduction). If the impact duration $T \ll \tau$, the second inequality is always satisfied, and the first one yields $\gamma \gg 0.1$ sec$^{-1}$ for $a = 1$ mm, $\delta = 3 \cdot 10^{-7}$ mm, and $\nu \ll 10^{13}$ sec$^{-1}$.

Consider a shock front much wider than the pore size and spacing. The movement of material and the charge distribution in the neighborhood of a spherical pore can then be assumed spherically symmetric. In that case we have from Eqs. (3)

$$\frac{\partial E_r}{\partial t} + \frac{v_r}{r^2}\frac{\partial}{\partial r}(r^2 E_r) + \alpha \frac{\partial}{\partial r} n (\nu_2 - \nu_1) + \frac{E_r}{\tau} = 0, \\ \alpha = e\delta^2/(\varepsilon\varepsilon_0), \ \tau = \varepsilon\varepsilon_0/(\sigma_1 + \sigma_2) \tag{4}$$

(r is the distance from the pore center, and $E_r$ and $v_r$ are the radial components of the vectors E and v). If we introduce the flux $\Phi = 4\pi r^2 E_r$ of E and transform to the Langrangian coordinates $r_0$ and t, Eq. (4) becomes

$$\frac{\partial \Phi}{\partial t} + \frac{\Phi}{\tau} + \alpha_0 r^2 \frac{\partial}{\partial r} n (\nu_2 - \nu_1) = 0, \quad \alpha_0 = \frac{\alpha}{4\pi}, \quad r = r(r_0, t). \tag{5}$$

The boundary conditions at the pore surface, at $r_0 = a_0$, are

$$J_r = -\partial\beta/\partial t, \ \beta = \varepsilon\varepsilon_0 E_r \ (r_0 = a_0 +). \tag{6}$$

A surface charge density β is assumed at the pore boundary. Furthermore, by symmetry, $E_r = 0$ within the pore. Using the equation for $J_r$ in Lagrangian coordinates to transform (6), we obtain the condition

$$\alpha \frac{\partial}{\partial r} n (\nu_2 - \nu_1) + \frac{E_r}{\tau} = -\left.\frac{\partial E_r}{\partial t}\right|_{r_0 = a_0 +},$$

which agrees with Eq. (5).

The solution of (5) with zero initial conditions is given by

$$\Phi(r_0, t) = \alpha_0 \int_0^t e^{(t'-t)/\tau} r^2 \frac{\partial}{\partial r} n (\nu_1 - \nu_2)\, dt', \quad r = r(r_0, t'). \tag{7}$$

In the case of an incompressible material, we have

$$r^3 - r_0^3 = a^3 - a_0^3, \quad \gamma = (a_0^3 - a^3)/r^3 \tag{8}$$

[$a = a(t)$ is the instantaneous pore radius].

The frequencies $\nu_i$ depend on the stress and temperature distributions near the pores. At pressures of ~1 GPa, at which cavities collapse in the shock wave, little heating occurs [4]. The stresses vary exponentially in the plastic zone around the pores. The difference $\nu_1 - \nu_2$ thus varies slowly with r and is primarily determined by the pressure amplitude in the shock wave. Using the expression for n in this approximation, we obtain from Eqs. (7) and (8) ($n_0 = 0$)

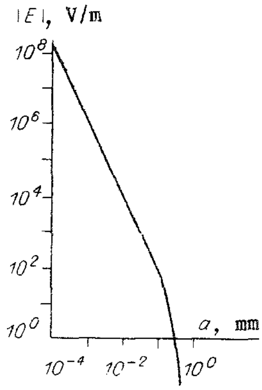


Fig. 1

$$\Phi(r_0, t) = 3\alpha_0 M (\nu_1 - \nu_2) \int_0^t e^{(t'-t)/\tau} \frac{(a^3 - a_0^3)\, dt'}{(r_0^3 + a^3 - a_0^3)^{2/3}}, \quad a = a(t'). \tag{9}$$

Most media are highly ductile in plastic flow, and as a result the pore radius decreases on the shock front smoothly, without oscillation [4]. The function $a^3 = a^3(t)$ can be approximated as

$$a^3 = a_*^3 + (a_0^3 - a_*^3)\, e^{-t/T} \quad (t \geq 0) \tag{10}$$

(T is the characteristic loading time, and $a_*$ is the final pore radius). The values of T and $a_*$ depend on the pressure amplitude in the shock wave.

Let us substitute (10) in (9). When $t \ll \tau$ the exponential in (9) is close to unity. Calculating the integral for this case, we get

$$E_r(r, t) = 9\alpha(\nu_1 - \nu_2) M T \psi(r, a)/r^2,$$

$$\psi = r_0 - r - \frac{(a_0^3 - a_*^3)}{b^2} \left[ \frac{1}{6} \ln \frac{(r_0 - b)^2 (r^2 + rb + b^2)}{(r - b)^2 (r_0^2 + r_0 b + b^2)} + \right. \tag{11}$$

$$\left. + \frac{1}{\sqrt{3}} \left( \operatorname{arctg} \frac{2r + b}{\sqrt{3}\, b} - \operatorname{arctg} \frac{2r_0 + b}{\sqrt{3}\, b} \right) \right], \quad b^3 = r^3 - a^3 + a_*^3,$$

where $a$ is given by (10) and $r_0$ by (8). If $t \gg \tau$, most of the integral in (9) accumulates at the upper limit. In that case

$$E_r(r, t) = 3\alpha(\nu_1 - \nu_2) M \tau (a_*^3 - a_0^3)(1 - e^{-t/\tau})/r^4. \tag{12}$$

The same follows from (3) if we set J = 0 and v = 0 for large times.

The linear function $n(\gamma)$ for point defects is restricted by the condition $n < n_* \sim 10^{28}$-$10^{29}$ m$^{-3}$ ($n_*$ is the lattice atom concentration). Thus, Eqs. (11) and (12) hold for $r = a$ if $a > a_c = a_0 (M/n_*)^{1/3} \sim 0.1 a_0$ ($M = 10^{25}$ m$^{-3}$). For further pore compression we can put $n = n_*$ and set $a = a_c$ in the function $\psi$. After expanding $\psi$ in the range $a_0 \geq a \geq a_*$, we obtain

$$E_r(a, t) = 3(15 + \sqrt{3})\, \alpha (\nu_2 - \nu_1)\, a_0 T M^{1/3} n_*^{2/3}/(4a^2). \tag{13}$$

We take as a lower limit for ionic crystals $\nu_2 - \nu_1 \sim 1$ sec$^{-1}$ (normal conditions). Then, given T = 1 msec, we get from (13) $E_r \sim 10^8$ V/m for $a \sim 0.5$ μm. The field $E_r$ at the pore surface is plotted against the pore radius with these parameters and $a_* = 0.1$ μm in Fig. 1. The electric field does not actually build up as $a \to 0$ because of defect recombination and dielectric breakdown (in ionic crystals $E_{br} \sim 10^8$ V/m).

The impact compression of porous solids at low pressures (~1 GPa) thus produces electric charge separation near cavities. Pore collapse intensifies the field and may cause electric breakdowns localized near the pores. In real solids the cavities are asymmetric, and therefore efficient electromagnetic radiators. Nonsymmetric pore compression in a shock wave produces electric polarization along the wave normal. This effect may account for the appearance of quasi-static electric fields in the near (nonwave) zone.

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# CAVITATION DYNAMICS IN REFLECTION OF A COMPRESSION PULSE FROM THE INTERFACE OF TWO MEDIA

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This paper is concerned with the reflection of a flat compression pulse, propagating in a condensed medium, from the surface of separation with a barrier, whose dynamic rigidity is low. This situation occurs in experiments on recording separation in low-strength substances – glycerol [1] or rubber [2]. In this case, as a result of interference of the incident and reflected rarefaction waves negative pressures are generated at some distance from the interface in the medium under study; these pressure gives rise to the appearance and growth of cavities – cavitation. The processes illustrated in Figs. 1 and 2, which show diagrams of the time t versus Langrangian coordinate h and the pressure p versus the mass velocity u of the material. The aim of this work is to determine the motion of the boundaries of the cavitation zone and the manifestation of this motion on the profile of the velocity of the contact boundary.

We study, in the acoustic approximation, cavitation in a medium whose tensile strength is equal to zero. We denote by $i_1 = \rho_{01}c_1$ and $i_2 = \rho_{02}c_2$ the dynamic rigidities of the material of interest and the barrier, respectively (ρ and c are the density and velocity of sound in the material). The incident compression pulse propagates along $C_+$ characteristics. After the shock wave emerges on the contact surface the reflected rarefaction wave, moving along $C_-$ characteristics, appears. The state of the particles of the material must satisfy conditions on both $C_+$ and $C_-$ characteristics.

Let the distribution of the velocity in the incident compression pulse have the form

$$u = u_0 - k(c_1t - h + H),\ u = 0 \text{ for } h - c_1t \leqslant H - u_0/k.$$

Here $u_0$ is the maximum value of the mass velocity and the coefficient k = const. Cavitation starts at t = τ in the section h = 0 (Fig. 1), where as a result of the interaction of the rarefaction waves the pressure first drops to zero. The left-hand boundary of the cavitation region is transported by the $C_-$ characteristic passing through this point (the line AB). After the reflected rarefaction wave encounters the end of the compression pulse at the point $t = u_0/2kc_1$, $h = H - u_0/2k$ the propagation of the cavitation zone to the left stops. From the conditions of compatibility of the states on the $C_+$ and $C_-$ characteristics (Fig. 2) it follows that the pressure p = 0 is reached at the time $\tau = H/c_1 = (u_0/kc_1)/((i_2)/(i_1 + i_2))$.

The change in the velocity and pressure on the contact boundary before information about the start of cavitation reaches it is described by the equations

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